

ORIGINAL PAPER

Open Access

# Memory-dependent derivative approach on magneto-thermoelastic transversely isotropic medium with two temperatures



Iqbal Kaur<sup>1\*</sup> , Parveen Lata<sup>2</sup> and Kulvinder Singh<sup>3</sup>

## Abstract

The aim of the present investigation is to examine the memory-dependent derivatives (MDD) in 2D transversely isotropic homogeneous magneto thermoelastic medium with two temperatures. The problem is solved using Laplace transforms and Fourier transform technique. In order to estimate the nature of the displacements, stresses and temperature distributions in the physical domain, an efficient approximate numerical inverse Fourier and Laplace transform technique is adopted. The distribution of displacements, temperature and stresses in the homogeneous medium in the context of generalized thermoelasticity using LS (Lord-Shulman) theory is discussed and obtained in analytical form. The effect of memory-dependent derivatives is represented graphically.

**Keywords:** Thermoelastic, Transversely isotropic, Magneto-thermoelastic, Memory-dependent derivative, Time delay, Kernel function, Lord-Shulman model

## Introduction

Magneto-thermoelasticity deals with the relations of the magnetic field, strain and temperature. It has wide applications such as geophysics, examining the effects of the earth's magnetic field on seismic waves, emission of electromagnetic radiations from nuclear devices and damping of acoustic waves in a magnetic field. In recent years, inspired by the successful applications of fractional calculus in diverse areas of engineering and physics, generalized thermoelasticity (GTE) models have been further comprehensive into temporal fractional ones to express memory dependence in heat conductive sense.

The MDD is defined in an integral form of a common derivative with a kernel function. The kernels in physical laws are important in many models that describe physical phenomena including the memory effect. Wang and Li (2011) introduced the concept of a MDD. Yu et al. (2014) introduced the MDD as an alternative of

fractional calculus into the rate of the heat flux in the Lord-Shulman (LS) theory of generalized thermoelasticity to represent memory dependence and recognized as a memory-dependent LS model. This innovative model might be useful to the fractional models owing to the following arguments. First, the new model is unique in its form, while the fractional-order models have different modifications (Riemann-Liouville, Caputo and other models). Second, the physical meaning of the new model is clearer due to the essence of the MDD definition. Third, the new model is depicted by integer-order differentials and integrals, which is more convenient in numerical calculation as compared to the fractional models. Finally, the kernel function and time delay of the MDD can be arbitrarily chosen; thus, the model is more flexible in applications than the fractional models, in which the significant variable is the fractional-order parameter (2016). Ezzat et al. (2014, 2015, 2016) discussed some solutions of one-dimensional problems obtained with the use of the memory-dependent LS model of generalized thermoelasticity.

\* Correspondence: [bawahanda@gmail.com](mailto:bawahanda@gmail.com)

<sup>1</sup>Department of Mathematics, Government College for Girls, Palwal, Kurukshetra, Haryana, India

Full list of author information is available at the end of the article

Ezzat et al. (2016) discussed a generalized model of two-temperature thermoelasticity theory with time delay and Kernel function and Taylor theorem with memory-dependent derivatives involving two temperatures. Ezzat and El-Bary (2017) applied the magneto-thermoelastic model to a one-dimensional thermal shock problem of functionally graded half space based on the MDD. Ezzat et al. (2017) proposed a mathematical model of electro-thermoelasticity for heat conduction with a MDD. Aldawody et al. (2018) proposed a new mathematical model of generalized magneto-thermo-viscoelasticity theories with MDD of dual-phase-lag heat conduction law. Despite this, several researchers worked on different theory of thermoelasticity such as Marin (1995, 2010), Mahmoud (2012), Riaz et al. (2019), Marin et al. (2013, 2016), Kumar and Chawla (2013), Sharma and Marin (2014), Kumar and Devi (2016), Bijarnia and Singh (2016), Othman et al. (2017), Ezzat and El-Barry (2017), Bhatti et al. (2019a, b, 2020), Youssef (2013, 2016), Lata et al. (2016), Othman and Marin (2017), Lata and Kaur (2019c, d, e) and Zhang et al. (2020).

In spite of these, not much work has been carried out in memory-dependent derivative approach for transversely isotropic magneto-thermoelastic medium with two temperatures. In this article, the memory-dependent derivatives (MDD) theory is revisited and it is adopted to analyse the effect of MDD in a homogeneous transversely isotropic magneto-thermoelastic solid. The problem is solved using Laplace transforms and Fourier transform technique. The components of displacement, conductive temperature and stress components in the homogeneous medium in the context of generalized thermoelasticity using LS (Lord-Shulman) theory is discussed and obtained in analytical form. The effect of memory-dependent derivatives is represented graphically.

**Basic equations**

Following Kumar et al. (2016), the simplified Maxwell’s linear equation of electrodynamics for a slowly moving and perfectly conducting elastic solid are

$$\text{curl } \vec{h} = \vec{j} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}, \tag{1}$$

$$\text{curl } \vec{E} = -\mu_0 \frac{\partial \vec{h}}{\partial t}, \tag{2}$$

$$\vec{E} = -\mu_0 \left( \frac{\partial \vec{u}}{\partial t} + \vec{H}_0 \right), \tag{3}$$

$$\text{div } \vec{h} = 0. \tag{4}$$

Maxwell stress components following Kumar et al. (2016) are given by

$$T_{ij} = \mu_0 (H_i h_j + H_j h_i - H_k h_k \delta_{ij}). \tag{5}$$

Equation of motion for a transversely isotropic thermoelastic medium and taking into account Lorentz force

$$t_{ij,j} + F_i = \rho \ddot{u}_i, \tag{6}$$

where  $F_i = \mu_0 (\vec{j} \times \vec{H}_0)_i$  are the components of Lorentz force. The constitutive relations for a transversely isotropic thermoelastic medium are given by

$$t_{ij} = C_{ijkl} e_{kl} - \beta_{ij} T. \tag{7}$$

and

$$\beta_{ij} = C_{ijk} \alpha_{ij}, \tag{8}$$

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), i, j = 1, 2, 3. \tag{9}$$

$$T = \phi - a_{ij} \phi_{,ij}, \tag{10}$$

$\alpha_{ij} = \alpha_i \delta_{ij}$ ,  $\beta_{ij} = \beta_i \delta_{ij}$ ,  $K_{ij} = K_i \delta_{ij}$ ,  $i$  is not summed.

Here,  $C_{ijkl}$  ( $C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}$ ) are elastic parameters and having symmetry ( $C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}$ ). The basis of these symmetries of  $C_{ijkl}$  is due to the following:

- i. The stress tensor is symmetric, which is only possible if ( $C_{ijkl} = C_{jilk}$ )
- ii. If a strain energy density exists for the material, the elastic stiffness tensor must satisfy  $C_{ijkl} = C_{klij}$
- iii. From stress tensor and elastic stiffness tensor, symmetries infer ( $C_{ijkl} = C_{ijlk}$ ) and  $C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}$

Following Bachher (2019), heat conduction equation for an anisotropic media is given by

$$K_{ij} \phi_{,ij} = (1 + \chi D_\chi) (\rho C_E T + \beta_{ij} T_0 \dot{e}_{ij}), \tag{11}$$

For the differentiable function  $f(t)$ , Wang and Li (2011) introduced the first-order MDD with respect to the time delay  $\chi > 0$  for a fixed time  $t$ :

$$D_\chi f(t) = \frac{1}{\chi} \int_{t-\chi}^t K(t-\xi) f'(\xi) d\xi, \tag{12}$$

The choice of the kernel function  $K(t - \xi)$  and the time delay parameter  $\chi$  is determined by the material properties. The kernel function  $K(t - \xi)$  is differentiable with respect to the variables  $t$  and  $\xi$ . The motivation for such a new definition is that it provides more insight into the memory effect (the instantaneous change rate depends on the past state) and also better physical meaning, which might be superior to the fractional models. This kind of the definition can reflect the memory effect on the delay interval  $[t - \chi, t]$ , which varies with time. They also suggested that the kernel form  $K(t$

$-\xi$ ) can also be chosen freely, e.g. as  $1, \xi - t + 1, [(\xi - t)/\chi + 1]^2$  and  $[(\xi - t)/\chi + 1]^{1/4}$ . The kernel function can be understood as the degree of the past effect on the present. Therefore, the forms  $[(\xi - t)/\chi + 1]^2$  and  $[(\xi - t)/\chi + 1]^{1/4}$  may be more practical because they are monotonic functions:  $K(t - \xi) = 0$  for the past time  $t - \chi$  and  $K(t - \xi) = 1$  for the present time  $t$ , i.e. it is easily concluded that the kernel function  $K(t - \xi)$  is a monotonic function increasing from zero to unity with time. The right side of the MDD definition given above can be understood as a mean value of  $f(\xi)$  on the past interval  $[t - \chi, t]$  with different weights. Generally, from the viewpoint of applications, the function  $K(t - \xi)$  should satisfy the inequality  $0 \leq K(t - \xi) < 1$  for  $\xi \in [t - \chi, t]$ . Therefore, the magnitude of the MDD  $D_\chi f(t)$  is usually smaller than that of the common derivative  $f'(t)$ . It can also be noted that the common derivative  $d/dt$  is the limit of  $D_\chi$  as  $\chi \rightarrow 0$ . Following Ezzat et al. (2014, 2015, 2016), the kernel function  $K(t - \xi)$  is taken here in the form

$$K(t - \xi) = 1 - \frac{2b}{\chi}(t - \xi) \frac{a^2}{\chi^2}(t - \xi)^2$$

$$= \begin{cases} 1 & a = 0, b = 0 \\ 1 + (\xi - t)/\chi & a = 0, b = 1/2 \\ \xi - t + 1 & a = 0, b = 1/\chi \\ [1 + (\xi - t)/\chi]^2 & a = 1, b = 1 \end{cases}$$

(13)

where  $a$  and  $b$  are constants. It should be also mentioned that the kernel in the fractional sense is singular, while that in the MDD model is non-singular. The kernel can be now simply considered a memory manager. The comma is further used to indicate the derivative with respect to the space variable, and the superimposed dot represents the time derivative.

**Method and solution of the problem**

We consider a homogeneous transversely isotropic magneto-thermoelastic medium initially at a uniform temperature  $T_0$ , permeated by an initial magnetic field  $\vec{H}_0 = (0, H_0, 0)$  acting along the  $y$ -axis. The rectangular Cartesian co-ordinate system  $(x, y, z)$  having origin on the surface ( $z = 0$ ) with  $z$ -axis pointing vertically into the medium is introduced. From the generalized Ohm's law

$$\vec{E} = -\mu_0 H_0 (-\ddot{w}, 0, \ddot{u})$$

(14)

$$\vec{j} = \left( -\frac{\partial Hy}{\partial z} - \epsilon_0 \dot{E}_x, 0, -\frac{\partial Hy}{\partial x} - \epsilon_0 \dot{E}_z \right)$$

(15)

$$\vec{F} = \left( \mu_0 H_0^2 \left( \frac{\partial e}{\partial x} - \epsilon_0 \mu_0 \ddot{u} \right), 0, \mu_0 H_0^2 \left( \frac{\partial e}{\partial z} - \epsilon_0 \mu_0 \ddot{w} \right) \right)$$

(16)

In addition, we consider the plane such that all particles on a line parallel to  $y$ -axis are equally displaced, so that the equations of displacement vector  $(u, v, w)$  and conductive temperature  $\phi$  for transversely isotropic thermoelastic solid are given by

$$u = u(x, z, t), v = 0, w = w(x, z, t) \text{ and } \phi = \phi(x, z, t)$$

(17)

Now, using the transformation on Eqs. (5) and (12) following Slaughter (2002) with the aid of (16) and (17) yields

$$C_{11} \frac{\partial^2 u}{\partial x^2} + C_{13} \frac{\partial^2 w}{\partial x \partial z} + C_{44} \left( \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x \partial z} \right) - \beta_1 \frac{\partial}{\partial x} \left\{ \phi - \left( a_1 \frac{\partial^2 \phi}{\partial x^2} + a_3 \frac{\partial^2 \phi}{\partial z^2} \right) \right\}$$

$$+ \mu_0 H_0^2 \left( \frac{\partial e}{\partial x} - \epsilon_0 \mu_0 \ddot{u} \right) = \rho \frac{\partial^2 u}{\partial t^2}$$

(18)

$$(C_{13} + C_{44}) \frac{\partial^2 u}{\partial x \partial z} + C_{44} \frac{\partial^2 w}{\partial x^2} + C_{33} \frac{\partial^2 w}{\partial z^2} - \beta_3 \frac{\partial}{\partial z} \left\{ \phi - \left( a_1 \frac{\partial^2 \phi}{\partial x^2} + a_3 \frac{\partial^2 \phi}{\partial z^2} \right) \right\}$$

$$+ \mu_0 H_0^2 \left( \frac{\partial e}{\partial z} - \epsilon_0 \mu_0 \ddot{w} \right) = \rho \frac{\partial^2 w}{\partial t^2}$$

(19)

$$K_1 \frac{\partial^2 \phi}{\partial x^2} + K_3 \frac{\partial^2 \phi}{\partial z^2} = (1 + \chi D_\chi) \left[ T_0 \left( \beta_1 \frac{\partial \dot{u}}{\partial x} + \beta_3 \frac{\partial \dot{w}}{\partial z} \right) + \rho C_E \left\{ \dot{\phi} - a_1 \frac{\partial^2 \phi}{\partial x^2} - a_3 \frac{\partial^2 \phi}{\partial z^2} \right\} \right]$$

(20)

where from (7)

$$\beta_1 = (C_{11} + C_{12})\alpha_1 + C_{13}\alpha_3, \beta_3 = 2C_{13}\alpha_1 + C_{33}\alpha_3, e = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}$$

And stress strain relations given by (5) after using (9) and (17) becomes

$$t_{11}(x, z, t) = C_{11} \frac{\partial u}{\partial x} + C_{13} \frac{\partial w}{\partial z} - \beta_1 \left\{ \phi - \left( a_1 \frac{\partial^2 \phi}{\partial x^2} + a_3 \frac{\partial^2 \phi}{\partial z^2} \right) \right\}$$

(21)

$$t_{22}(x, z, t) = c_{12} \frac{\partial u}{\partial x} + c_{13} \frac{\partial w}{\partial z} - \beta_1 \left\{ \phi - \left( a_1 \frac{\partial^2 \phi}{\partial x^2} + a_3 \frac{\partial^2 \phi}{\partial z^2} \right) \right\}$$

(22)

$$t_{33}(x, z, t) = C_{13} \frac{\partial u}{\partial x} + C_{33} \frac{\partial w}{\partial z} - \beta_3 \left\{ \phi - \left( a_1 \frac{\partial^2 \phi}{\partial x^2} + a_3 \frac{\partial^2 \phi}{\partial z^2} \right) \right\}$$

(23)

$$t_{13}(x, z, t) = C_{44} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)$$

(24)

$$t_{12}(x, z, t) = t_{23}(x, z, t) = 0$$

(25)

We assume that medium is initially at rest. The undisturbed state is maintained at reference temperature  $T_0$ . Then, we have the initial and regularity conditions given by

$$\begin{aligned} u(x, z, 0) = 0 = \dot{u}(x, z, 0), w(x, z, 0) = 0 = \dot{w}(x, z, 0), \phi(x, z, 0) \\ = 0 = \dot{\phi}(x, z, 0) \text{ for } z \geq 0, -\infty < x < \infty, u(x, z, t) \\ = w(x, z, t) = \phi(x, z, t) = 0 \text{ for } t > 0 \text{ when } z \rightarrow \infty. \end{aligned}$$

To simplify the solution, the following dimensionless quantities are used:

$$\begin{aligned} (x', z') &= \frac{1}{L}(x, z), (u', w') = \frac{\rho C_1^2}{L \beta_1 T_0}(u, w), (a'_1, a'_3) \\ &= \frac{1}{L^2}(a_1, a_3), \rho C_1^2 = C_{11} \phi' = \frac{\phi}{T_0}, (t'_{11}, t'_{13}, t'_{33}) \\ &= \frac{1}{\beta_1 T_0}(t_{11}, t_{13}, t_{33}), t' = \frac{C_1}{L}t. \end{aligned} \tag{26}$$

Making use of dimensionless quantities defined by (26) in Eqs. (18)–(20), after suppressing the primes, yields

$$\begin{aligned} (1 + \delta_5) \frac{\partial^2 u}{\partial x^2} + (\delta_4 + \delta_5) \frac{\partial^2 w}{\partial x \partial z} + \delta_2 \left( \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x \partial z} \right) - \frac{\partial}{\partial x} \left\{ \phi - \left( a_1 \frac{\partial^2 \phi}{\partial x^2} + a_3 \frac{\partial^2 \phi}{\partial z^2} \right) \right\} \\ = \left( \frac{\epsilon_0 \mu_0^2 H_0^2}{\rho} + 1 \right) \frac{\partial^2 u}{\partial t^2}, \end{aligned} \tag{27}$$

$$\begin{aligned} (\delta_1 + \delta_5) \frac{\partial^2 u}{\partial x \partial z} + \delta_2 \frac{\partial^2 w}{\partial x^2} + (\delta_3 + \delta_5) \frac{\partial^2 w}{\partial z^2} - \frac{\beta_3}{\beta_1} \frac{\partial}{\partial z} \left\{ \phi - \left( a_1 \frac{\partial^2 \phi}{\partial x^2} + a_3 \frac{\partial^2 \phi}{\partial z^2} \right) \right\} \\ = \left( \frac{\epsilon_0 \mu_0^2 H_0^2}{\rho} + 1 \right) \frac{\partial^2 w}{\partial t^2}, \end{aligned} \tag{28}$$

$$K_1 \frac{\partial^2 \phi}{\partial x^2} + K_3 \frac{\partial^2 \phi}{\partial z^2} = (1 + \chi D_\chi)$$

$$\left[ \frac{\partial}{\partial t} \left( \delta_6 \frac{\partial u}{\partial x} + \delta_8 \frac{\partial w}{\partial z} \right) + \delta_7 \frac{\partial}{\partial t} \left\{ \phi - a_1 \frac{\partial^2 \phi}{\partial x^2} - a_3 \frac{\partial^2 \phi}{\partial z^2} \right\} \right], \tag{29}$$

where

$$\begin{aligned} \delta_1 &= \frac{C_{13} + C_{44}}{C_{11}}, \delta_2 = \frac{C_{44}}{C_{11}}, \delta_3 = \frac{C_{33}}{C_{11}}, \delta_4 = \frac{C_{13}}{C_{11}}, \delta_5 \\ &= \frac{\beta_1 T_0 \mu_0 H_0^2}{L \rho^2 C_1^4}, \delta_6 = \frac{L \beta_1^2 T_0}{\rho C_1}, \delta_8 = \frac{L \beta_1 \beta_3 T_0}{\rho C_1}, \delta_7 \\ &= \rho C_E C_1 L. \end{aligned}$$

Laplace transforms is defined by

$$\mathcal{L}[f(x, z, t)] = \int_0^\infty e^{-st} f(x, z, t) dt = \tilde{f}(x, z, s). \tag{30}$$

with basic properties

$$L\left(\frac{\partial f}{\partial t}\right) = s\tilde{f}(x, z, s) - f(x, z, 0), \tag{31}$$

$$L\left(\frac{\partial^2 f}{\partial t^2}\right) = s^2\tilde{f}(x, z, s) - sf(x, z, 0) - \left(\frac{\partial f}{\partial t}\right)_{t=0}. \tag{32}$$

Fourier transforms of a function  $f$  with respect to variable  $x$  with  $\xi$  as a Fourier transform variable is defined by

$$\mathcal{F}\{\tilde{f}(x, z, s)\} = \hat{f}(\xi, z, s) = \int_{-\infty}^\infty \tilde{f}(x, z, s) e^{i\xi x} dx. \tag{33}$$

With basic properties: if  $f(x)$  and first  $(n - 1)$  derivatives of  $f(x)$  vanish identically as  $x \rightarrow \pm \infty$ , then

$$\mathcal{F}\left\{\frac{\partial^n \tilde{f}(x, z, s)}{\partial x^n}\right\} = (i\xi)^n \hat{f}(\xi, z, s) \tag{34}$$

Applying Laplace and Fourier transforms defined by (31)–(34) on Eqs. (24)–(29) yields

$$\begin{aligned} - (1 + \delta_5) \xi^2 \hat{u} + (\delta_4 + \delta_5) i \xi D \hat{w} \\ + \delta_2 (D^2 \hat{u} + i \xi D \hat{w}) - i \xi \left\{ \hat{\phi} - (-a_1 \xi^2 \hat{\phi} + a_3 D^2 \hat{\phi}) \right\} \\ = \left( \frac{\epsilon_0 \mu_0^2 H_0^2}{\rho} + 1 \right) s^2 \hat{u}, \end{aligned} \tag{35}$$

$$\begin{aligned} (\delta_1 + \delta_5) i \xi D \hat{u} - \delta_2 \xi^2 \hat{w} \\ + (\delta_3 + \delta_5) D^2 \hat{w} - \frac{\beta_3}{\beta_1} D \left\{ \hat{\phi} - (-a_1 \xi^2 \hat{\phi} + a_3 D^2 \hat{\phi}) \right\} \\ = \left( \frac{\epsilon_0 \mu_0^2 H_0^2}{\rho} + 1 \right) s^2 \hat{w}, \end{aligned} \tag{36}$$

$$\begin{aligned} - K_1 \xi^2 \hat{\phi} + K_3 D^2 \hat{\phi} = (1 + G) \\ [s(\delta_6 i \xi \hat{u} + \delta_8 D \hat{w}) + \delta_7 s \left\{ \hat{\phi} - (-a_1 \xi^2 \hat{\phi} + a_3 D^2 \hat{\phi}) \right\}], \end{aligned} \tag{37}$$

where

$$D = \frac{d}{dz}, G = \frac{\tau_0}{\chi} \left[ (1 - e^{-sx}) \left( 1 - \frac{2b}{\chi s} + \frac{2a^2}{\chi^2 s^2} \right) - \left( a^2 - 2b + \frac{2a^2}{\chi s} \right) e^{-sx} \right].$$

These equations on further simplifications become

$$\begin{aligned} \left[ - (1 + \delta_5) \xi^2 - \left( \frac{\epsilon_0 \mu_0^2 H_0^2}{\rho} + 1 \right) s^2 + \delta_2 D^2 \right] \hat{u}(\xi, z, s) \\ + [(\delta_4 + \delta_5 + \delta_2) i \xi D] \hat{w}(\xi, z, s) \\ + (-i \xi) [1 + a_1 \xi^2 - a_3 D^2] \hat{\phi}(\xi, z, s) = 0, \end{aligned} \tag{38}$$

$$[(\delta_1 + \delta_5)i\xi D]\hat{u}(\xi, z, s) + \left[-\delta_2\xi^2 + (\delta_3 + \delta_5)D^2 - \left(\frac{\epsilon_0\mu_0^2 H_0^2}{\rho} + 1\right)s^2\right]\hat{u}(\xi, z, s) - \frac{\beta_3}{\beta_1}D[1 + a_1\xi^2 - a_3D^2]\hat{\phi}(\xi, z, s) = 0, \tag{39}$$

$$[(1 + G)s\delta_6 i\xi]\hat{u}(\xi, z, s) + [(1 + G)s\delta_8 D]\hat{w}(\xi, z, s) + [-K_1\xi^2 + K_3D^2 + (1 + G)\delta_7 s(1 + a_1\xi^2 - a_3D^2)]\hat{\phi}(\xi, z, s) = 0, \tag{40}$$

The above equations can be written as

$$[\zeta_1 + \delta_2 D^2]\hat{u}(\xi, z, s) + [\zeta_2 D]\hat{w}(\xi, z, s) + [\zeta_3 + \zeta_4 D^2]\hat{\phi}(\xi, z, s) = 0, \tag{41}$$

$$[\zeta_5 D]\hat{u}(\xi, z, s) + [\zeta_6 + \zeta_7 D^2]\hat{w}(\xi, z, s) + [\zeta_8 D + \zeta_9 D^3]\hat{\phi}(\xi, z, s) = 0, \tag{42}$$

$$[\zeta_{10}]\hat{u}(\xi, z, s) + [\zeta_{11}]\hat{w}(\xi, z, s) + [\zeta_{12} + \zeta_{13} D^2]\hat{\phi}(\xi, z, s) = 0, \tag{43}$$

Equations (41)–(43) will have a non-trivial solution if determinant of coefficient matrix of  $(\hat{u}, \hat{w}, \hat{\phi})$  vanishes, and thus we obtain the characteristic equation as

$$(AD^6 + BD^4 + CD^2 + E)(\hat{u}, \hat{w}, \hat{\phi}) = 0, \tag{44}$$

where

$$D = \frac{d}{dz}, A = -\delta_2\zeta_{11}\zeta_9 + \zeta_{13}\delta_2\zeta_7, \\ B = \delta_2\zeta_6\zeta_{13} + \zeta_7\zeta_1\zeta_{13} + \delta_2\zeta_7\zeta_{12} - \delta_2\zeta_{11}\zeta_8 - \zeta_1\zeta_9\zeta_{11} - \zeta_5\delta_2\zeta_{13} + \delta_2\zeta_{10}\zeta_9 + \zeta_{11}\zeta_4\zeta_5 - \zeta_{10}\zeta_7\zeta_4, \\ C = \zeta_{13}\zeta_1\zeta_6 + \delta_2\zeta_6\zeta_{12} + \zeta_1\zeta_7\zeta_{12} - \zeta_1\zeta_{11}\zeta_8 + \zeta_4\zeta_8\zeta_9 - \zeta_5\delta_2\zeta_{12} - \zeta_{10}\delta_2\zeta_8 + \zeta_{11}\zeta_3\zeta_5 - \zeta_3\zeta_7\zeta_{10}, \\ E = \zeta_{12}\zeta_1\zeta_6 - \zeta_{10}\zeta_6\zeta_3, \\ \zeta_1 = -(1 + \delta_5)\xi^2 - \left(\frac{\epsilon_0\mu_0^2 H_0^2}{\rho} + 1\right)s^2, \zeta_2 = (\delta_4 + \delta_5 + \delta_2)i\xi, \zeta_3 = -i\xi(1 + a_1\xi^2), \\ \zeta_4 = a_3i\xi, \zeta_5 = (\delta_1 + \delta_5)i\xi, \zeta_6 = -\delta_2\xi^2 - \left(\frac{\epsilon_0\mu_0^2 H_0^2}{\rho} + 1\right)s^2, \zeta_7 = (\delta_3 + \delta_5), \\ \zeta_8 = -(1 + a_1\xi^2)\frac{\beta_3}{\beta_1}, \zeta_9 = \frac{\beta_3}{\beta_1}a_3, \zeta_{10} = [(1 + G)s\delta_6 i\xi], \zeta_{11} = (1 + G)s\delta_8, \\ \zeta_{12} = [-K_1\xi^2 + (1 + G)\delta_7 s(1 + a_1\xi^2)], \zeta_{13} = K_3 - (1 + G)\delta_7 s a_3$$

The roots of Eq. (44) are  $\pm \lambda_i$  ( $i = 1, 2, 3$ ). The solution of Eqs. (41)–(43) satisfying the radiation condition that  $(\hat{u}, \hat{w}, \hat{\phi}) \rightarrow 0$  as  $z \rightarrow \infty$  can be written as

$$\hat{u}(\xi, z, s) = \sum_{j=1}^3 A_j e^{-\lambda_j z}, \tag{45}$$

$$\hat{w}(\xi, z, s) = \sum_{j=1}^3 d_j A_j e^{-\lambda_j z}, \tag{46}$$

$$\hat{\phi}(\xi, z, s) = \sum_{j=1}^3 l_j A_j e^{-\lambda_j z}, \tag{47}$$

where  $A_j, j = 1, 2, 3$  being undetermined constants and  $d_j$  and  $l_j$  are given by

$$d_j = \frac{\delta_2\zeta_{13}\lambda_j^4 + (\zeta_{13}\zeta_1 - \zeta_{10}\zeta_4 + \delta_2\zeta_{12})\lambda_j^2 + \zeta_1\zeta_{12} - \zeta_{10}\zeta_3}{(\zeta_{13}\zeta_7 - \zeta_1\zeta_9)\lambda_j^4 + (\zeta_{13}\zeta_6 + \zeta_{12}\zeta_7 - \zeta_1\zeta_8)\lambda_j^2 + \zeta_{12}\zeta_6}, \tag{48}$$

$$l_j = \frac{\delta_2\zeta_7\lambda_j^4 + (\delta_2\zeta_6 + \zeta_1\zeta_7 - \delta_2\zeta_5)\lambda_j^2 + \zeta_6\zeta_1}{(\zeta_{13}\zeta_7 - \zeta_1\zeta_9)\lambda_j^4 + (\zeta_{13}\zeta_6 + \zeta_{12}\zeta_7 - \zeta_1\zeta_8)\lambda_j^2 + \zeta_{12}\zeta_6}. \tag{49}$$

$$\hat{t}_{11}(\xi, z, s) = i\xi\hat{u}(\xi, z, s) + \delta_4 D\hat{w}(\xi, z, s) - \frac{\beta_3}{\beta_1}[1 + a_1\xi^2 - a_3D^2]\hat{\phi}(\xi, z, s), \tag{50}$$

$$\hat{t}_{33}(\xi, z, s) = \delta_4 i\xi\hat{u}(\xi, z, s) + \delta_3 D\hat{w}(\xi, z, s) - \frac{\beta_3}{\beta_1}[1 + a_1\xi^2 - a_3D^2]\hat{\phi}(\xi, z, s), \tag{51}$$

$$\hat{t}_{13}(\xi, z, s) = \delta_2(D\hat{u}(\xi, z, s) + i\xi\hat{w}(\xi, z, s)). \tag{52}$$

Thus, using (45)–(47) in Eqs. (50)–(52) gives

$$\hat{t}_{11}(\xi, z, s) = i\xi \sum_{j=1}^3 A_j e^{-\lambda_j z} + \delta_4 D \sum_{j=1}^3 d_j A_j e^{-\lambda_j z} - [1 + a_1\xi^2 - a_3D^2] \sum_{j=1}^3 l_j A_j e^{-\lambda_j z}, \tag{53}$$

$$\hat{t}_{33}(\xi, z, s) = \delta_4 i\xi \sum_{j=1}^3 A_j e^{-\lambda_j z} + \delta_3 D \sum_{j=1}^3 d_j A_j e^{-\lambda_j z} - \frac{\beta_3}{\beta_1} [1 + a_1\xi^2 - a_3D^2] \sum_{j=1}^3 l_j A_j e^{-\lambda_j z}, \tag{54}$$

$$\hat{t}_{13}(\xi, z, s) = \delta_2 \left( D \sum_{j=1}^3 A_j e^{-\lambda_j z} + i\xi \sum_{j=1}^3 d_j A_j e^{-\lambda_j z} \right). \tag{55}$$

which on further simplification gives

$$\hat{t}_{11}(\xi, z, s) = ((i\xi - \delta_4\lambda_1 d_1 - ([1 + a_1\xi^2 - a_3\lambda_1^2]l_1))e^{-\lambda_1 z} A_1 + (i\xi - \delta_4\lambda_2 d_2 - ([1 + a_1\xi^2 - a_3\lambda_2^2]l_2))e^{-\lambda_2 z} A_2 + (i\xi - \delta_4\lambda_3 d_3 - ([1 + a_1\xi^2 - a_3\lambda_3^2]l_3))e^{-\lambda_3 z} A_3), \tag{56}$$

$$\hat{t}_{33}(\xi, z, s) = \left( (\delta_4 i\xi - \delta_3\lambda_1 d_1 - \frac{\beta_3}{\beta_1}([1 + a_1\xi^2 - a_3\lambda_1^2]l_1))e^{-\lambda_1 z} A_1 + (\delta_4 i\xi - \delta_3\lambda_2 d_2 - \frac{\beta_3}{\beta_1}([1 + a_1\xi^2 - a_3\lambda_2^2]l_2))e^{-\lambda_2 z} A_2 + (\delta_4 i\xi - \delta_3\lambda_3 d_3 - \frac{\beta_3}{\beta_1}([1 + a_1\xi^2 - a_3\lambda_3^2]l_3))e^{-\lambda_3 z} A_3 \right), \tag{57}$$

$$\hat{t}_{13}(\xi, z, s) = (-\lambda_1 + i\xi d_1)\delta_2 A_1 e^{-\lambda_1 z} + (-\lambda_2 + i\xi d_2)\delta_2 A_2 e^{-\lambda_2 z} + (-\lambda_3 + i\xi d_3)\delta_2 A_3 e^{-\lambda_3 z} \tag{58}$$

which can be further simplified as

$$\hat{t}_{11}(\xi, z, s) = (S_1 A_1 + S_2 A_2 + S_3 A_3) \tag{59}$$

$$\hat{t}_{33}(\xi, z, s) = (M_1 A_1 + M_2 A_2 + M_3 A_3) \tag{60}$$

$$\hat{t}_{13}(\xi, z, s) = (N_1 A_1 + N_2 A_2 + N_3 A_3) \tag{61}$$

where

$$S_j = \left\{ i\xi - \delta_4 d_j \lambda_j - l_j \left[ (1 + a_1 \xi^2) - a_3 \lambda_j^2 \right] \right\} e^{-\lambda_j z} \tag{62}$$

$$N_j = \delta_2 (-\lambda_j + i\xi d_j) e^{-\lambda_j z}, \tag{63}$$

$$M_j = \left\{ \delta_4 i\xi - \delta_3 d_j \lambda_j - \frac{\beta_3}{\beta_1} l_j \left[ (1 + a_1 \xi^2) - a_3 \lambda_j^2 \right] \right\} e^{-\lambda_j z}, \tag{64}$$

**Boundary conditions**

The appropriate boundary conditions for the thermally insulated boundaries  $z = 0$  are

$$t_{33}(x, z, t) = -F_1 \psi_1(x) H(t), \tag{65}$$

$$t_{31}(x, z, t) = -F_2 \psi_2(x) H(t), \tag{66}$$

$$\frac{\partial \phi}{\partial z}(x, z, t) + h\phi = 0. \tag{67}$$

where  $h \rightarrow 0$  corresponds to thermally insulated surface and  $h \rightarrow \infty$  corresponds to isothermal surface,  $F_1$  and  $F_2$  are the magnitude of the forces applied and  $\psi_1(x)$  and  $\psi_2(x)$  specify the vertical and horizontal load distribution function along  $x$ -axis.

Applying dimensionless conditions (26) and suppressing primes and then applying the Laplace and Fourier transform defined by (30) and (33) on the boundary conditions (65)–(67) and using (47), (60) and (61), the three equations in three variables  $A_1, A_2, A_3$  are

$$\hat{t}_{33}(\xi, 0, s) = M_1 A_1 + M_2 A_2 + M_3 A_3 = \frac{-F_1 \psi_1(\xi)}{s}, \tag{68}$$

$$\hat{t}_{13}(\xi, 0, s) = (N_1 A_1 + N_2 A_2 + N_3 A_3) = \frac{-F_2 \psi_2(\xi)}{s}, \tag{69}$$

$$D\hat{\phi}(\xi, 0, s) + h\hat{\phi}(\xi, 0, s) = (R_1 A_1 + R_2 A_2 + R_3 A_3) = 0, \tag{70}$$

where  $R_j = -\lambda_j l_j + h l_j$ , note at  $z = 0, e^{-\lambda_j z} = 1$ , in (62–64) for values of  $S_j, M_j, N_j$  and  $D = \frac{\partial}{\partial z}$ .

**Case I: Thermally insulated boundaries**

When  $h = 0$  solving (68)–(70), the values of  $A_1, A_2, A_3$  are obtained as

$$A_1 = \frac{-F_1 \psi_1(\xi) \Lambda_{11} + F_2 \psi_2(\xi) \Lambda_{21}}{\Lambda_s}, \tag{71}$$

$$A_2 = \frac{F_1 \psi_1(\xi) \Lambda_{12} - F_2 \psi_2(\xi) \Lambda_{22}}{\Lambda_s}, \tag{72}$$

$$A_3 = \frac{F_1 \psi_1(\xi) \Lambda_{13} - F_2 \psi_2(\xi) \Lambda_{23}}{\Lambda_s}, \tag{73}$$

where

$$\begin{aligned} \Lambda_{11} &= -N_2 R_3 + R_2 N_3, \\ \Lambda_{12} &= N_1 R_3 - R_1 N_3, \\ \Lambda_{13} &= -N_1 R_2 + R_1 N_2, \\ \Lambda_{21} &= M_2 R_3 - R_2 M_3, \\ \Lambda_{22} &= -M_1 R_3 + R_1 M_3, \\ \Lambda_{23} &= M_1 R_2 - R_1 M_2, \\ \Lambda &= -M_1 \Lambda_{11} - M_2 \Lambda_{12} - M_3 \Lambda_{13}, \\ R_j &= -\lambda_j l_j. \end{aligned}$$

The components of displacement, conductive temperature, normal stress and tangential stress are obtained from (45) to (47) and (59) to (61) by putting the values of  $A_1, A_2, A_3$  from (71) to (73) as

$$\hat{u} = \frac{F_1 \hat{\psi}_1(\xi)}{s\Lambda} \left[ \sum_{i=1}^3 \Lambda_{1i} e^{-\lambda_i z} \right] + \frac{F_2 \hat{\psi}_2(\xi)}{s\Lambda} \left[ \sum_{i=1}^3 \Lambda_{2i} e^{-\lambda_i z} \right], \tag{74}$$

$$\hat{w} = \frac{F_1 \hat{\psi}_1(\xi)}{s\Lambda} \left[ \sum_{i=1}^3 d_i \Lambda_{1i} e^{-\lambda_i z} \right] + \frac{F_2 \hat{\psi}_2(\xi)}{s\Lambda} \left[ \sum_{i=1}^3 d_i \Lambda_{2i} e^{-\lambda_i z} \right], \tag{75}$$

$$\hat{\phi} = \frac{F_1 \hat{\psi}_1(\xi)}{s\Lambda} \left[ \sum_{i=1}^3 l_i \Lambda_{1i} e^{-\lambda_i z} \right] + \frac{F_2 \hat{\psi}_2(\xi)}{s\Lambda} \left[ \sum_{i=1}^3 l_i \Lambda_{2i} e^{-\lambda_i z} \right], \tag{76}$$

$$\hat{t}_{11} = \frac{F_1 \hat{\psi}_1(\xi)}{s\Lambda} \left[ \sum_{i=1}^3 S_i \Lambda_{1i} e^{-\lambda_i z} \right] + \frac{F_2 \hat{\psi}_2(\xi)}{s\Lambda} \left[ \sum_{i=1}^3 S_i \Lambda_{2i} e^{-\lambda_i z} \right], \tag{77}$$

$$\hat{t}_{13} = \frac{F_1 \hat{\psi}_1(\xi)}{s\Lambda} \left[ \sum_{i=1}^3 N_i \Lambda_{1i} e^{-\lambda_i z} \right] + \frac{F_2 \hat{\psi}_2(\xi)}{s\Lambda} \left[ \sum_{i=1}^3 N_i \Lambda_{2i} e^{-\lambda_i z} \right], \tag{78}$$

$$\hat{t}_{33} = \frac{F_1 \hat{\psi}_1(\xi)}{s\Lambda} \left[ \sum_{i=1}^3 M_i \Lambda_{1i} e^{-\lambda_i z} \right] + \frac{F_2 \hat{\psi}_2(\xi)}{s\Lambda} \left[ \sum_{i=1}^3 M_i \Lambda_{2i} e^{-\lambda_i z} \right], \tag{79}$$

**Case II: Isothermal boundaries**

When  $h \rightarrow \infty$ , solving (68)–(70), using Cramer’s rule, the values of  $A_1, A_2, A_3$  are obtained as

$$A_1 = \frac{-F_1\psi_1(\xi)\Lambda_{11}^* + F_2\psi_2(\xi)\Lambda_{21}^*}{\Lambda^*s}, \tag{80}$$

$$A_2 = \frac{F_1\psi_1(\xi)\Lambda_{12}^* - F_2\psi_2(\xi)\Lambda_{22}^*}{\Lambda^*s}, \tag{81}$$

$$A_3 = \frac{F_1\psi_1(\xi)\Lambda_{13}^* - F_2\psi_2(\xi)\Lambda_{23}^*}{\Lambda^*s}, \tag{82}$$

where

$$\begin{aligned} \Lambda_{11}^* &= -N_2R_3^* + R_2^*N_3, \\ \Lambda_{12}^* &= N_1R_3^* - R_1^*N_3, \\ \Lambda_{13}^* &= -N_1R_2^* + R_1^*N_2, \\ \Lambda_{21}^* &= M_2R_3^* - R_2^*M_3, \\ \Lambda_{22}^* &= -M_1R_3^* + R_1^*M_3, \\ \Lambda_{23}^* &= M_1R_2^* - R_1^*M_2, \\ \Lambda^* &= -M_1\Lambda_{11}^* - M_2\Lambda_{12}^* - M_3\Lambda_{13}^*, \end{aligned}$$

where  $R_j^* = l_j$ .

The components of displacement, conductive temperature, normal stress and tangential stress are obtained from (45) to (47) and (59) to (61) by putting the values of  $A_1, A_2, A_3$  from (80) to (82) as

$$\hat{u} = \frac{F_1\hat{\psi}_1(\xi)}{s\Lambda^*} \left[ \sum_{i=1}^3 \Lambda_{1i}^* e^{-\lambda_i z} \right] + \frac{F_2\hat{\psi}_2(\xi)}{s\Lambda^*} \left[ \sum_{i=1}^3 \Lambda_{2i}^* e^{-\lambda_i z} \right], \tag{83}$$

$$\hat{w} = \frac{F_1\hat{\psi}_1(\xi)}{s\Lambda^*} \left[ \sum_{i=1}^3 l_i \Lambda_{1i}^* e^{-\lambda_i z} \right] + \frac{F_2\hat{\psi}_2(\xi)}{s\Lambda^*} \left[ \sum_{i=1}^3 d_i \Lambda_{2i}^* e^{-\lambda_i z} \right], \tag{84}$$

$$\hat{\phi} = \frac{F_1\hat{\psi}_1(\xi)}{s\Lambda^*} \left[ \sum_{i=1}^3 l_i \Lambda_{1i}^* e^{-\lambda_i z} \right] + \frac{F_2\hat{\psi}_2(\xi)}{s\Lambda^*} \left[ \sum_{i=1}^3 l_i \Lambda_{2i}^* e^{-\lambda_i z} \right], \tag{85}$$

$$\hat{t}_{11} = \frac{F_1\hat{\psi}_1(\xi)}{s\Lambda^*} \left[ \sum_{i=1}^3 S_i \Lambda_{1i}^* e^{-\lambda_i z} \right] + \frac{F_2\hat{\psi}_2(\xi)}{s\Lambda^*} \left[ \sum_{i=1}^3 S_i \Lambda_{2i}^* e^{-\lambda_i z} \right], \tag{86}$$

$$\hat{t}_{13} = \frac{F_1\hat{\psi}_1(\xi)}{s\Lambda^*} \left[ \sum_{i=1}^3 N_i \Lambda_{1i}^* e^{-\lambda_i z} \right] + \frac{F_2\hat{\psi}_2(\xi)}{s\Lambda^*} \left[ \sum_{i=1}^3 N_i \Lambda_{2i}^* e^{-\lambda_i z} \right], \tag{87}$$

$$\hat{t}_{33} = \frac{F_1\hat{\psi}_1(\xi)}{s\Lambda^*} \left[ \sum_{i=1}^3 M_i \Lambda_{1i}^* e^{-\lambda_i z} \right] + \frac{F_2\hat{\psi}_2(\xi)}{s\Lambda^*} \left[ \sum_{i=1}^3 M_i \Lambda_{2i}^* e^{-\lambda_i z} \right], \tag{88}$$

### Applications

We consider a normal line load  $F_1$  per unit length acting in the positive  $z$ -axis on the plane boundary  $z = 0$  along the  $y$ -axis and a tangential load  $F_2$  per unit length, acting at the origin in the positive  $x$ -axis. Suppose an inclined load,  $F_0$  per unit length is acting on the  $y$ -axis and its inclination with  $z$ -axis is  $\theta$  (see Fig. 1), we have

$$F_1 = F_0 \cos\theta \text{ and } F_2 = F_0 \sin\theta. \tag{89}$$

### Special cases

#### Concentrated force

The solution due to concentrated normal force on the half space is obtained by setting

$$\psi_1(x) = \delta(x), \psi_2(x) = \delta(x), \tag{90}$$

where  $\delta(x)$  is Dirac delta function.

Applying Fourier transform defined by (33) on (90) yields

$$\hat{\psi}_1(\xi) = 1, \hat{\psi}_2(\xi) = 1. \tag{91}$$

For case I, using (91) in Eqs. (74)–(79) and for case II, using (91) in Eqs. (83)–(88), the components of displacement, stress and conductive temperature are obtained for case I and case II, respectively.

#### Uniformly distributed force

The solution due to uniformly distributed force applied on the half space is obtained by setting

$$\psi_1(x), \psi_2(x) = \begin{cases} 1 & \text{if } |x| \leq m \\ 0 & \text{if } |x| > m \end{cases} \tag{92}$$

The Fourier transforms of  $\psi_1(x)$  and  $\psi_2(x)$  with respect to the pair  $(x, \xi)$  for the case of a uniform strip load of non-dimensional width  $2m$  applied at origin of coordinate system  $x = z = 0$  in the dimensionless form after suppressing the primes becomes

$$\hat{\psi}_1(\xi) = \hat{\psi}_2(\xi) = \left\{ \frac{2 \sin(\xi m)}{\xi} \right\}, \xi \neq 0. \tag{93}$$

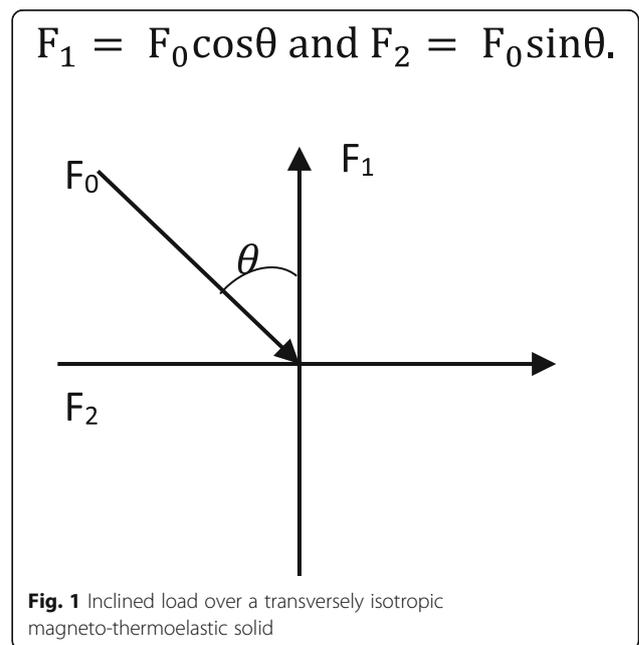


Fig. 1 Inclined load over a transversely isotropic magneto-thermoelastic solid

For case I, using (93) in Eqs. (74)–(79) and for case II, using (93) in Eqs. (83)–(88), the components of displacement, stress and conductive temperature are obtained for case I and case II.

**Linearly distributed force**

The solution due to linearly distributed force applied on the half space is obtained by setting

$$\{\psi_1(x), \psi_2(x)\} = \begin{cases} 1 - \frac{|x|}{m} & \text{if } |x| \leq m \\ 0 & \text{if } |x| > m \end{cases} \quad (94)$$

Here, 2m is the width of the strip load, and applying the transform defined by (33) on (94), we get

$$\hat{\psi}_1(\xi) = \hat{\psi}_2(\xi) = \left\{ \frac{2(1 - \cos(\xi m))}{\xi^2 m} \right\}, \xi \neq 0. \quad (95)$$

For case I, using (95) in Eqs. (74)–(79) and for case II, using (95) in Eqs. (83)–(88), the components of displacement, stress and conductive temperature are obtained for case I and case II, respectively.

**Inversion of the transformation**

To find the solution of the problem in physical domain and to invert the transforms in Eqs (74)–(79) and for case II in Eqs. (83)–(88), here, the displacement components, normal and tangential stresses and conductive temperature are functions of z, the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form  $\hat{f}(\xi, z, s)$ . To find the function  $\tilde{f}(x, z, t)$  in the physical domain, we first invert the Fourier transform using

$$\begin{aligned} \tilde{f}(x, z, s) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\xi x} \hat{f}(\xi, z, s) d\xi \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} |\cos(\xi x) f_e - i \sin(\xi x) f_o| d\xi, \end{aligned} \quad (96)$$

where  $f_e$  and  $f_o$  are respectively the odd and even parts of  $\hat{f}(\xi, z, s)$ . We obtain Fourier inverse transform by replacing s by ω in (96). Following Honig and Hirdes (1984), the Laplace transform function  $\tilde{f}(x, z, s)$  can be inverted to  $f(x, z, t)$  for problem I by

$$f(x, z, t) = \frac{1}{2\pi i} \int_{e^{-i\infty}}^{e^{+i\infty}} \tilde{f}(x, z, s) e^{-st} ds. \quad (97)$$

The last step is to calculate the integral in Eq. (97). The method for evaluating this integral is described in Press (1986).

**Numerical results and discussion**

To demonstrate the theoretical results and effect of memory-dependent derivatives, the physical data for cobalt material, which is transversely isotropic, is taken from Kumar et al. (2016) given as

$$\begin{aligned} C_{11} &= 3.07 \times 10^{11} Nm^{-2}, C_{33} \\ &= 3.581 \times 10^{11} Nm^{-2}, C_{13} \\ &= 1.027 \times 10^{10} Nm^{-2}, C_{44} \\ &= 1.510 \times 10^{11} Nm^{-2}, \beta_1 \\ &= 7.04 \times 10^6 Nm^{-2} deg^{-1}, \beta_3 \\ &= 6.90 \times 10^6 Nm^{-2} deg^{-1}, \rho \\ &= 8.836 \times 10^3 Kgm^{-3}, C_E \\ &= 4.27 \times 10^2 jKg^{-1} deg^{-1}, K_1 \\ &= 0.690 \times 10^2 Wm^{-1} Kdeg^{-1}, K_3 \\ &= 0.690 \times 10^2 Wm^{-1} K^{-1}, K_1^* \\ &= 1.313 \times 10^2 Wsec, K_3^* = 1.54 \times 10^2 Wsec, T_0 \\ &= 298K, H_0 = 1 Jm^{-1} nb^{-1}, \epsilon_0 \\ &= 8.838 \times 10^{-12} Fm^{-1}, L = 1. \end{aligned}$$

The values of rotation Ω and magnetic effect  $H_0$  are taken as 0.5 and 10, respectively. The software FORTRAN has been used to determine the components of displacement, stress and conductive temperature. A comparison has been made to show the effect of kernel function of MDD on the various quantities.

1. The solid line with square symbol represents  $a = 0.0, b = 0.0, K(t - \xi) = 1$ ,
2. The dashed line with circle symbol represents  $a = 0.0, b = \frac{1}{2}, K(t - \xi) = 1 + (\xi - t)/\chi$ ,
3. The dotted line with triangle symbol represents  $a = 0.0, b = \frac{1}{\chi}, K(t - \xi) = \xi - t + 1$ ,
4. The dash-dotted line with diamond symbol represents  $a = 1, b = 1, K(t - \xi) = [1 + (\xi - t)/\chi]^2$

Figures 2, 3, 4, 5, 6 and 7 shows the variations of the displacement components (u and w), conductive temperature φ and stress components (t<sub>11</sub>, t<sub>13</sub> and t<sub>33</sub>) for a transversely isotropic magneto-thermoelastic medium with concentrated force and with combined effects of rotation, two temperatures with different kernel function K(t - ξ), respectively. The displacement components (u and w), conductive temperature φ and stress components (t<sub>11</sub> and t<sub>13</sub>) illustrate the opposite behaviour for the kernel function  $a = 0.0, b = \frac{1}{\chi}, K(t - \xi) = \xi - t + 1$  but with other kernel functions shows the same pattern. However, stress component t<sub>33</sub> illustrate the opposite behaviour for the kernel function  $[1 + (\xi - t)/\chi]^2$  but with other kernel functions shows the same pattern. These components vary (increases or decreases) during the initial range of distance near

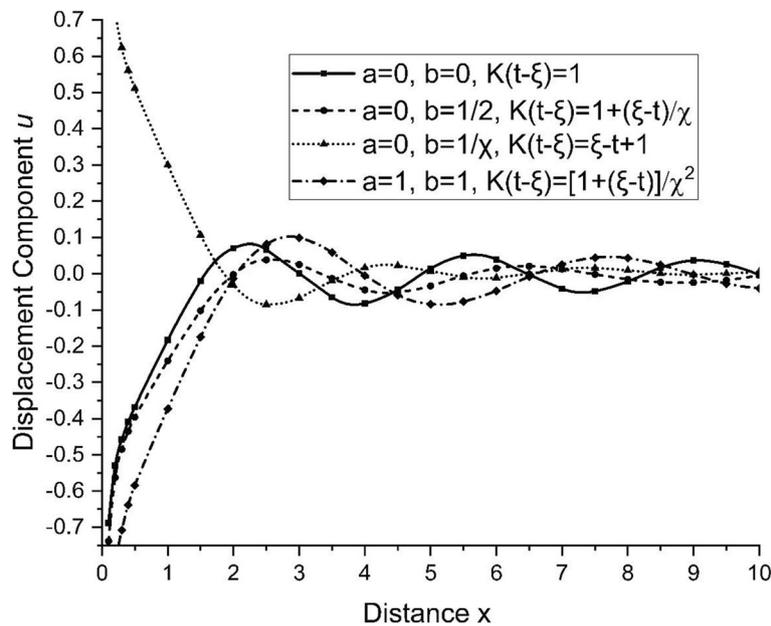


Fig. 2 Variation of the displacement component  $u$  with distance  $x$

the loading surface of the inclined load and follow a small oscillatory pattern for the rest of the range of distance.

**Conclusion**

From the above study, the following is observed:

- Displacement components ( $u$  and  $w$ ), conductive temperature  $\phi$  and stress components ( $t_{11}$ ,  $t_{13}$  and

$t_{33}$ ) for a transversely isotropic magneto-thermoelastic medium with concentrated force and with combined effects of two-temperature model of thermoelasticity with different kernel functions of MDD, respectively

- In order to estimate the nature of the displacements, stresses and temperature distributions in the physical domain, an efficient approximate numerical inverse Fourier and Laplace transform technique is adopted.

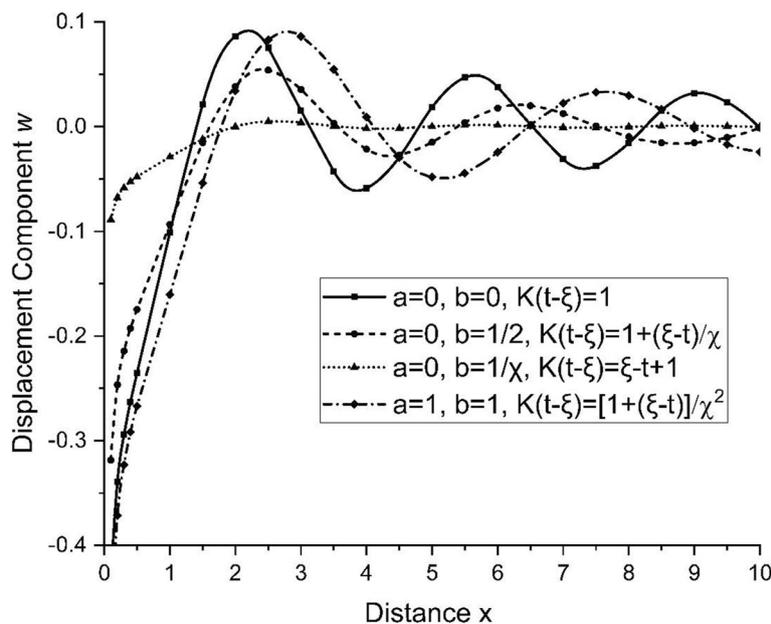


Fig. 3 Variation of the displacement component  $w$  with distance  $x$

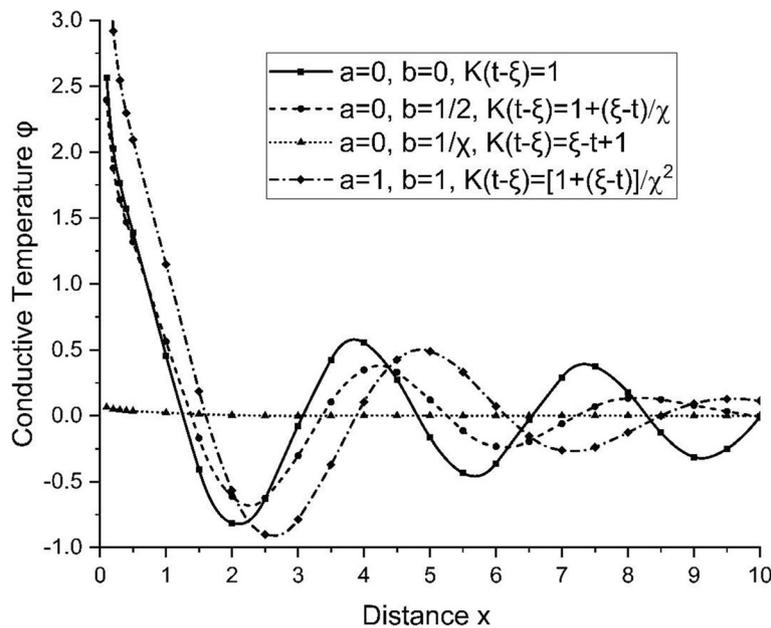


Fig. 4 Variation of the conductive temperature  $\phi$  with distance  $x$

- Moreover, the magnetic effect of two temperatures, rotation, and the angle of inclination of the applied load plays a key part in the deformation of all the physical quantities.
- $K(t - \xi) = [1 + (\xi - t)]/\chi^2$  shows the more oscillatory nature for the displacement components and stress components.
- The result gives the inspiration to study magneto-thermoelastic materials with memory-dependent derivatives as an innovative domain of applicable thermoelastic solids.
- The shape of curves shows the impact of Kernel function on the body and fulfils the purpose of the study.

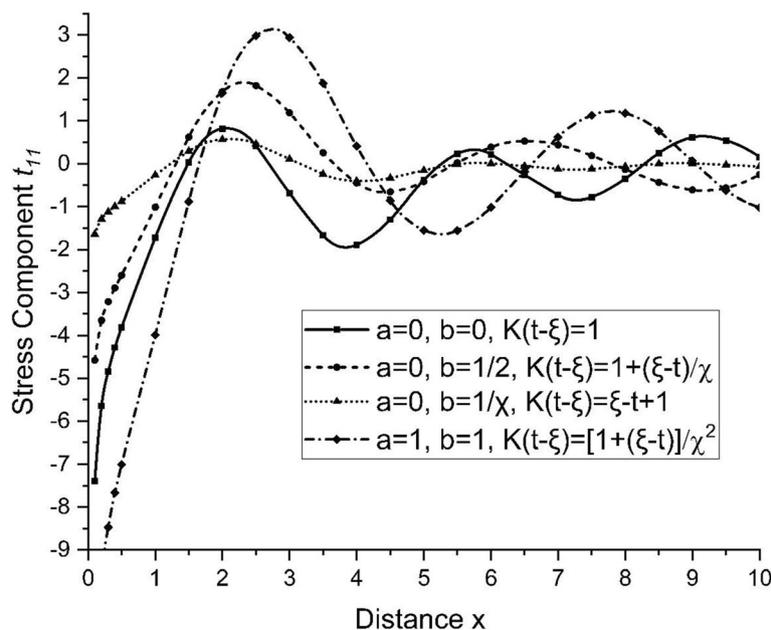


Fig. 5 Variation of the stress component  $t_{11}$  with distance  $x$

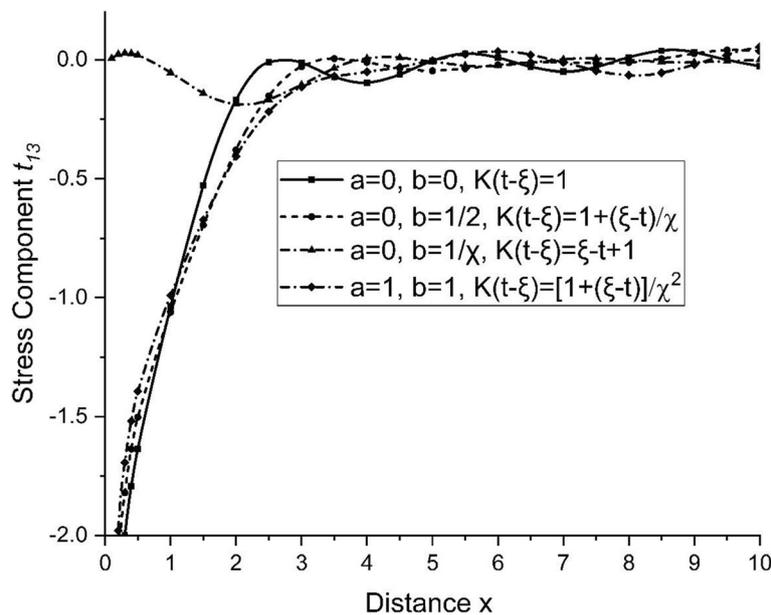


Fig. 6 Variation of the stress component  $t_{13}$  with distance  $x$

- The outcomes of this research are extremely helpful in the 2-D problem with dynamic response of memory-dependent derivatives in transversely isotropic magneto-thermoelastic medium with two temperatures which is advantageous to successful applications of memory dependence in heat conductive sense.

**Nomenclature**

- $\delta_{ij}$  Kronecker delta
- $C_{ijkl}$  Elastic parameters
- $\beta_{ij}$  Thermal elastic coupling tensor
- $T$  Absolute temperature
- $T_0$  Reference temperature
- $\phi$  conductive temperature

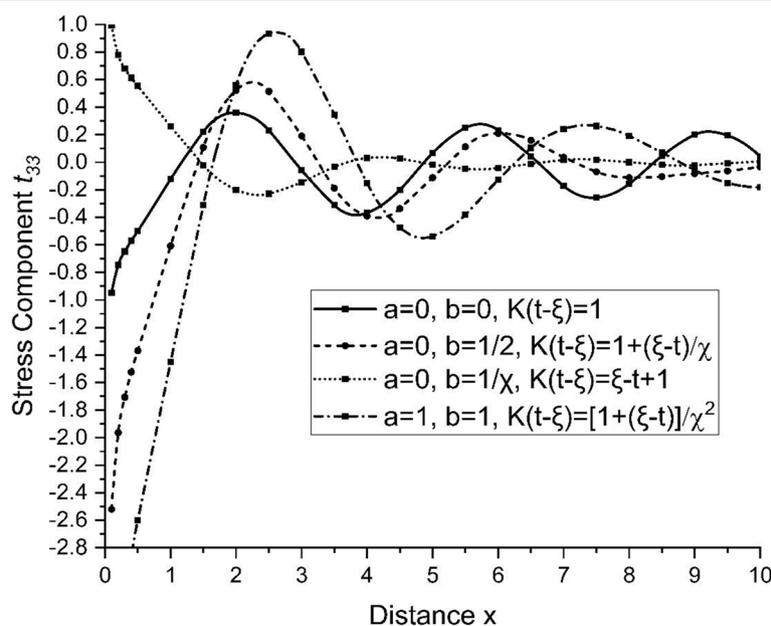


Fig. 7 Variation of the stress component  $t_{33}$  with distance  $x$

$t_{ij}$  Stress tensors  
 $e_{ij}$  Strain tensors  
 $u_i$  Components of displacement  
 $\rho$  Medium density  
 $C_E$  Specific heat  
 $a_{ij}$  Two-temperature parameters  
 $\alpha_{ij}$  Linear thermal expansion coefficient  
 $K_{ij}$  Thermal conductivity  
 $T_{ij}$  Maxwell stress tensor  
 $\chi$  Time delay  
 $\mu_0$  Magnetic permeability  
 $K(t - \xi)$  Kernel function  
 $\vec{u}$  Displacement vector  
 $\vec{H}_0$  Magnetic field intensity vector  
 $\vec{j}$  Current density vector  
 $F_i$  Components of Lorentz force  
 $\tau_0$  Relaxation time  
 $\epsilon_0$  Electric permeability  
 $\delta(t)$  Dirac's delta function  
 $\vec{h}$  Induced magnetic field vector  
 $\vec{E}$  Induced electric field vector

#### Acknowledgements

Not applicable.

#### Authors' contributions

The work is carried by the corresponding author under the guidance and supervision of Dr. Parveen Lata and Dr. Kulvinder Singh. The author(s) read and approved the final manuscript.

#### Funding

No fund/grant/scholarship has been taken for the research work.

#### Availability of data and materials

For the numerical results, cobalt material has been taken for thermoelastic material from Kumar et al. (2016).

#### Competing interests

The authors declare that they have no competing interests.

#### Author details

<sup>1</sup>Department of Mathematics, Government College for Girls, Palwal, Kurukshetra, Haryana, India. <sup>2</sup>Department of Basic and Applied Sciences, Punjabi University, Patiala, Punjab, India. <sup>3</sup>Kurukshetra University, Kurukshetra, Haryana, India.

Received: 22 April 2020 Accepted: 4 November 2020

Published online: 03 December 2020

#### References

- Aldawody, D. A., Hendy, M. H., & Ezzat, M. A. (2018). On dual-phase-lag magneto-thermo-viscoelasticity theory with memory-dependent derivative. *Microsystem Technologies*. <https://doi.org/10.1007/s00542-018-4194-6>.
- Bachher, M. (2019). Plane harmonic waves in thermoelastic materials with a memory-dependent derivative. *Journal of Applied Mechanics and Technical Physics*, 60(1), 123–131.
- Bhatti, M. M., Ellahi, R., Zeeshan, A., Marin, M., & Ijaz, N. (2019a). Numerical study of heat transfer and Hall current impact on peristaltic propulsion of particle-fluid suspension with compliant wall properties. *Modern Physics Letters B*, 35(35). <https://doi.org/10.1142/S0217984919504396>.
- Bhatti, M. M., Shahid, A., Abbas, T., Alamri, S. Z., & Ellahi, R. (2020). Study of activation energy on the movement of gyrotactic microorganism in a magnetized nanofluids past a porous plate. *Processes*, 8(3), 328–348. <https://doi.org/10.3390/pr8030328>.
- Bhatti, M. M., Yousif, M. A., Mishra, S. R., & Shahid, A. (2019b). Simultaneous influence of thermo-diffusion and diffusion-thermo on non-Newtonian hyperbolic tangent magnetised nanofluid with Hall current through a nonlinear stretching surface. *Pramana*, 93(6), 88. <https://doi.org/10.1007/s12043-019-1850-z>.
- Bijarnia, R., & Singh, B. (2016). Propagation of plane waves in a rotating transversely isotropic two temperature generalized thermoelastic solid half-space with voids. *International Journal of Applied Mechanics and Engineering*, 21(1), 285–301. <https://doi.org/10.1515/ijame-2016-0018>.
- Ezzat, M., & El-Barry, A. A. (2017). Fractional magneto-thermoelastic materials with phase-lag Green-Naghdi theories. *Steel and Composite Structures*, 24(3), 297–307. <https://doi.org/10.12989/scs.2017.24.3.297>.
- Ezzat, M. A., & El-Bary, A. A. (2017). A functionally graded magneto-thermoelastic half space with memory-dependent derivatives heat transfer. *Steel and Composite Structures*, 25(2), 177–186.
- Ezzat, M. A., El-Karamany, A. S., & El-Bary, A. A. (2014). Generalized thermo-viscoelasticity with memory dependent derivatives. *International Journal of Mechanical Sciences*, 89, 470–475.
- Ezzat, M. A., El-Karamany, A. S., & El-Bary, A. A. (2015). A novel magneto thermoelasticity theory with memory dependent derivative. *Journal of Electromagnetic Waves and Applications*, 29(8), 1018–1031.
- Ezzat, M. A., El-Karamany, A. S., & El-Bary, A. A. (2016). Generalized thermoelasticity with memory-dependent derivatives involving two-temperatures. *Mechanics of Advanced Materials and Structures*, 23, 545–553.
- Ezzat, M. A., Karamany, A. S., & El-Bary, A. (2017). Thermoelectric viscoelastic materials with memory-dependent derivative. *Smart Structures and Systems, An Int'l Journal*, 19(5), 539–551.
- Honig, G., & Hirdes, U. (1984). A method for the numerical inversion of Laplace transform. *Journal of Computational and Applied Mathematics*, 10, 113–132.
- Kumar, R., & Chawla, V. (2013). Reflection and refraction of plane wave at the interface between elastic and thermoelastic media with three-phase-lag model. *International Communications in Heat and Mass Transfer (Elsevier)*, 48, 53–60.
- Kumar, R., & Devi, S. (2016). Plane waves and fundamental solution in a modified couple stress generalized thermoelastic with three-phase-lag model. *Multidiscipline Modeling in Materials and Structures (Emerald)*, 12(4), 693–711.
- Kumar, R., Sharma, N., & Lata, P. (2016). Thermomechanical interactions in transversely isotropic magnetothermoelastic medium with vacuum and with and without energy dissipation with combined effects of rotation, vacuum and two temperatures. *Applied Mathematical Modelling*, 40(13-14), 6560–6575.
- Lata, P., & Kaur, I. (2019c). Plane wave propagation in transversely isotropic magnetothermoelastic rotating medium with fractional order generalized heat transfer. *Structural Monitoring and Maintenance*, 6(3), 191–218. <https://doi.org/10.12989/smm.2019.6.3.191>.
- Lata, P., & Kaur, I. (2019d). Axisymmetric thermomechanical analysis of transversely isotropic magneto thermoelastic solid due to time-harmonic sources. *Coupled Systems Mechanics*, 8(5), 415–437. <https://doi.org/10.12989/csm.2019.8.5.415>.
- Lata, P., & Kaur, I. (2019e). Effect of rotation and inclined load on transversely isotropic magneto thermoelastic solid. *Structural Engineering and Mechanics*, 70(2), 245–255. <https://doi.org/10.12989/sem.2019.70.2.245>.
- Lata, P., Kumar, R., & Sharma, N. (2016). Plane waves in an anisotropic thermoelastic. *Steel and Composite Structures*, 22(3), 567–587. <https://doi.org/10.12989/scs.2016.22.3.567>.
- Mahmoud, S. (2012). Influence of rotation and generalized magneto-thermoelastic on Rayleigh waves in a granular medium under effect of initial stress and gravity field. *Meccanica, Springer*, 47, 1561–1579. <https://doi.org/10.1007/s11012-011-9535-9>.
- Marin, M. (1995). On existence and uniqueness in thermoelasticity of micropolar bodies. *Comptes Rendus De L Academie*, 321, 475–480.
- Marin, M. (2010). Some estimates on vibrations in thermoelasticity of dipolar bodies. *Journal of Vibration and Control: SAGE Journals*, 16(1), 33–47.
- Marin, M., Agarwal, R., & Mahmoud, S. (2013). Nonsimple material problems addressed by the Lagrange's identity. *Boundary Value Problems*, 2013(135), 1–14.
- Marin, M., Craciun, E. M., & Pop, N. (2016). Considerations on mixed initial-boundary value problems for micropolar porous bodies. *Dynamic Systems and Applications*, 25(1-2), 175–196.
- Othman, M., & Marin, M. (2017). Effect of thermal loading due to laser pulse on thermoelastic porous medium under G-N theory. *Results in Physics*, 7, 3863–3872.

- Othman, M. I., Abo-Dahab, S. M., & Alsebaey, S. O. N. (2017). Reflection of plane waves from a rotating magneto-thermoelastic medium with two-temperature and initial stress under three theories. *Mechanics and Mechanical Engineering*, 21(2), 217–232.
- Press, W. T. (1986). *Numerical recipes in Fortran*. Cambridge: Cambridge University Press.
- Riaz, A., Ellahi, R., Bhatti, M. M., & Marin, M. (2019). Study of heat and mass transfer in the Eyring–Powell model of fluid propagating peristaltically through a rectangular compliant channel. *Heat Transfer Research*, 50(16), 1539–1560. <https://doi.org/10.1615/heattransres.2019025622>.
- Sharma, K., & Marin, M. (2014). Reflection and transmission of waves from imperfect boundary between two heat conducting micropolar thermoelastic solids. *Analele Universitatii "Ovidius" Constanta-Seria Matematica*, 22(2), 151–175. <https://doi.org/10.2478/auom-2014-0040>.
- Slaughter, W. S. (2002). *The linearised theory of elasticity*. Birkhäuser Basel, Boston. <https://doi.org/10.1007/978-1-4612-0093-2>.
- Wang, J., & Li, H. (2011). Surpassing the fractional derivative: Concept of the memory-dependent derivative. *Computers & Mathematics with Applications*, 62(3), 1562–1567.
- Youssef, H. M. (2013). State-space approach to two-temperature generalized thermoelasticity without energy dissipation of medium subjected to moving heat source. *Applied Mathematics and Mechanics*, 34(1), 63–74. <https://doi.org/10.1007/s10483-013-1653-7>.
- Youssef, H. M. (2016). Theory of generalized thermoelasticity with fractional order strain. *Journal of Vibration and Control*, 22(18), 3840–3857. <https://doi.org/10.1177/1077546314566837>.
- Yu, Y.-J., Hu, W., & Tian, X.-G. (2014). A novel generalized thermoelasticity model based on memory-dependent derivative. *International Journal of Engineering Science*, 81, 123–134.
- Zhang, L., Arain, M. B., Bhatti, M. M., Zeeshan, A., & Hal-Sulami, H. (2020). Effects of magnetic Reynolds number on swimming of gyrotactic microorganisms between rotating circular plates filled with nanofluids. *Applied Mathematics and Mechanics*, 41(4), 637–654. <https://doi.org/10.1007/s10483-020-2599-7>.

## Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Submit your manuscript to a SpringerOpen<sup>®</sup> journal and benefit from:

- Convenient online submission
- Rigorous peer review
- Open access: articles freely available online
- High visibility within the field
- Retaining the copyright to your article

---

Submit your next manuscript at ► [springeropen.com](https://www.springeropen.com)

---